**Glider Transient Model (Unpowered)**

Sum of forces in x-direction (see Fig. 1):

(W/g)\*dVx/dt = (-1)\*(D\*cos g + L\*sin g)

dVx/dt = ax = (-1)\*g\*[ (D/W)\*cos g + (L/W)\*sin g) ] Equ. 1

Sum of forces in y-direction (see Fig. 1):

(W/g)\*dVy/dt = (L\*cos g - D\*sin g - W)

dVy/dt = ay = (-1)\*g\*[ 1 + (D/W)\*sin g - (L/W)\*cos g) ] Equ. 2

Assumptions

-Still air, no head or cross wind

-Equations good for climb, cruise and descent

-trajectory is for a point mass at c.g., i.e. a/c is longitudinally trimmed, no rotation about center-of gravity (c.g.).

-motion in x-y plane only, no rotation about c.g. in x-z or y-z

planes .

-Aircraft pitch angle + wing incidence angle = constant

L=Lift

D=Drag

W=Weight

Y axis

X axis

g = flight path angle

Figure 1 - Force Balance in accelerated 2D flight

Need expressions for L/W and D/W;

1. **L/W = Lift-to-Weight ratio**

By definition,

CL = Lift Coefficient = L/(q\*S) = n\*W\*cos g/(q\*S) Equ. 3

Where;

q (lb/ft2) = dynamic pressure = 0.5\*rho\*(V2) Equ. 4

rho (slugs/ft3) = ambient density of air, (slugs/ft3) Equ. 5

V (ft/sec) = flight speed Equ. 6

n = load factor, (dimensionless, # of g’s) Equ. 7

Assuming small flight path angles, i.e. g<10 deg, and no turning (n~1)

L/W = CL\*q/(W/S) Equ. 8

Develop an expression for CL (see Figure 2).

y = m\*x + b (simple straight line)

CL = CLa\*a + CLo (linear model) Equ. 9

CL = Lift Coefficient

CLa = lift curve slope, 1/deg

CLo = Lift coefficient at a=0 deg

aoL = angle-of-attack at CL=0

CL, (-)

a, (deg)

x intercept, aoL

y intercept, CLo

Figure 2 - Lift Curve (linear model)

Slope= dCL/da = CLa

At CL=0 … 0 = CLa\*aoL + CLo, therefore CLo = (-1)\*CLa\*aoL Equ. 10

Sub Equ. 10 into Equ. 9

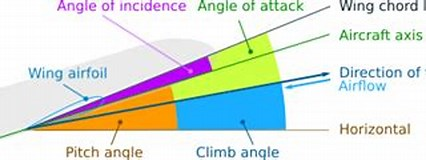
CL = CLa\*a – CLa\*aoL Equ. 11

CL = CLa\*(a-aoL) Equ. 12

CL = CLa\*(iw-g-aoL) (a=iw+p-g, see Figure 3 below) Equ. 13

Sub Equ. 13 into Equ. 8

L/W = CLa\*(iw-g-aoL)\*q/(W/S) Equ. 14



**Figure 3 - Angles**

Chord line of wing airfoil

Fuselage

centerline

iw, angle of incidence +

p, aircraft pitch angle

1. **On the ground and level flight**

Horizontal

Datum

Iw+p

a

g

**b) Climbing flight**

1. **D/W = Drag-to-Weight ratio**

By definition,

Drag Coefficient = CD = D/(q\*S) Equ. 15

D/W = CD\*q\*S/W = CD\*q / (W/S) Equ. 16

Develop expression for CD (see Figure 4)

CD = CDo+k\*CL2 Equ. 17

Use Equ. 17 in Equ. 16

D/W = (CDo+(k\*CL2))\*(q/(W/S)) Equ. 18

Figure 4 – Drag Polar

CD

CL

CDo

CD = Drag Coefficient (simple parabolic)

CD = CDo + k\*CL2

Where;

k=induced drag factor = 1/pi\*A\*e

A = Wing aspect ratio

e = Oswald’s efficiency factor

CDo = Parasite drag coefficient

**Summary**

Equations 1, 2, 14 and 18 define the system of equations t be solved.

dVx/dt = ax = (-1)\*g\*[ (D/W)\*cos g + (L/W)\*sin g) ] Equ. 1

dVy/dt = ay = (-1)\*g\*[ 1 + (D/W)\*sin g - (L/W)\*cos g) ] Equ. 2

L/W = CLa\*(iw-g-aoL)\*q/(W/S) Equ. 14

D/W = (CDo+(k\*CL2))\*(q/(W/S)) Equ. 18

Also,

q = 0.5\*rho\*(V2)

g = arctan (Vx/Vy)

Equations 1 and 2 are two (2) simultaneous differential equations. One way to solve these is with a numerical solution technique called the 4th Order Runge-Kutta Method. See this link for some general information on the method, <https://en.wikipedia.org/wiki/Runge–Kutta_methods>

The method uses a combination of various slopes to estimate the value of a function at a later time based on a step size.

See Excel Sheet for model